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Tri-valued fuzzy soft ideals of hemi-rings and decision making

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Abstract
The goal of this work is to present the notions of tri-value fuzzy soft ideal, tri-value equivalent fuzzy soft ideals of hemi-rings. Some properties and characteristics of them are given. Some examples are given. Lastly, we examine aggregative tri-valued fuzzy ideals of hemi-rings created on selection taking.

Keywords: fuzzy set; fuzzy soft set; tri-valued fuzzy soft ideal; hemiring.

1 Introduction

Handling difficulties in various regions of applied mathematics and sciences, we can find that more applicable in semi-rings. They involve more essential part intake in optimization areas, automata theory, graph theory, and so on. Most semi-rings in the part of structure theory, and generally not meet the ideals of semi-rings with normally ring ideals, hence limited usage of ideals in semi-rings several authors mean a semi ring along with a "0" and along with a commutative " $\odot$ " we said to be semiring hemiring. The h-ideals in hemi-rings were systematically examined by [3] and apply the h-ideals, he has proven a few equivalents ring of hemi-rings, Jun *et al.* [12] make known to the idea of fuzzy h-ideals in hemi-rings and provided a few connected properties. In investigating Zhan [5] the h-hemi-regular hemi-rings. Zhan [13] gave more these articles [11] gave a few generalized fuzzy h-ideals of Hemi-rings. Zhan and Shum [4] have read intuitionistic fuzzy h-ideals of hemi-rings. Dudek [10] gave the essential results on hemi-rings. Mathematical show-off and operation of a few types of doubts have come to be a gradually more crucial problem in answering difficult troubles springing up in a huge collection of extents likewise medicine, engineering, economics, etc. Molodtsov [2] initially presented the soft set concept as a common mathematical tool for uncertainness. Therefore a few authors gave operation of soft sets and use [8] a moment ago, [1] a few authors find fuzzy soft sets along by way of parameterizations, such as parameterized soft sets. [9] gave intuitionistic fuzzy parameterized soft sets. Zadeh [7] gave the concept of fuzzy set, Atanassov [6] presented the idea of the intuitionistic fuzzy set. In this work, we initiated the idea of tri-valued fuzzy soft ideals of hemi-rings and argue a few connected results. Also using two operator maximum ($\vee$), minimum ($\wedge$) operator. As a final topic, we saw a few examples to display the techniques can be effectively useful to a few uncertain problems.

2 Preliminaries

Let $(S, \oplus, \odot)$ be a semiring involving a non-null set S along with two operation on S is said to be "$\oplus$" and "$\odot$" such that $(S, \oplus)$ and $(S, \odot)$ be a semigroups and fulfills the distributive property (Here $\oplus, \odot$ be the usual addition and multiplication).

$$l(m \oplus n) = lm \oplus ln, \quad (l \oplus m)n = ln \oplus mn, \quad \forall\ l, m, n \in S \tag{2.1}$$

Let $(S, \oplus, \odot)$ be a semi-ring and let zero be a semi-ring of $(S, \oplus, \odot)$ that mean the element $0 \in S$ such that $0 \odot l = l \odot 0$ and $0 \oplus l = l \oplus 0, \forall\ l \in S$. $(H, \oplus)$ is said to hemi-ring the following conditions are fulfilled a semiring along with zero, commutative semi-group $(S, \oplus)$. For simply we write lm for $l \odot m(l, m \in H)$.

A subhemiring of a hemi-ring $H \subseteq L$ of H is closed under "$\oplus$" and "$\odot$". A subset L of H is said to be left right ideal of H, if L is closed under "$\oplus$", $L \supseteq HL$ and $L \supseteq LH$. A subset L is said to be an ideal, if it left and right ideals.

A subhemiring L of H is said to be an h-subhemiring of H, if for any $a, c \in H$ and $l, m \in L$, and $a \oplus l \oplus c = m \oplus c \Rightarrow l \in L$.

Let C be the common universal set and H is hemi-ring, F be the factor set. Hence the power set be $P(C)$, and $L, M, N \subseteq F$.

Definition 2.1. [7] Let C be a common universal set. A fuzzy set L over C is a set well-defined by a function φ_L be the mapping

$$\varphi_L = C \to [0, 1] \tag{2.2}$$

Here φ_L is said to be the membership value of L and the value $\varphi_L(l)$ is said to be the grade of membership of $\varphi \in C$. The value denotes the degree of $l \in L$. Hence a fuzzy set L over C can be symbolized as

$$L = \{(l, \varphi_L(l)) : l \in C, \varphi_L(l) \in [0, l]\} \tag{2.3}$$

Here the set of all fuzzy sets over L symbolized by $L(C)$.

Definition 2.2. A tri-valued fuzzy set (TVFS) L over C is stated as below

$$L = \{(l, \varphi_L(l), \theta_L(l)) : l \in C\} \tag{2.4}$$

where $0 \leq \varphi_L(l) \leq \varphi_L(l) \leq 1$ this value is membership and non membership value. It also additionally $\forall\, l \in C, C = \{(l,1,0) : l \in C\}$, void set $=\{(l,0,1) : l \in C\}$ are tri-value fuzzy common Universal and tri-Value fuzzy void set respectively.

Definition 2.3. (a) Let φ be a fuzzy set H is called fuzzy LRI(left-right ideal) of H. If the below property satisfies $\varphi(l \oplus m) \geq \inf\{\varphi(l), \varphi(m)\}, \quad \varphi(lm) \geq \varphi(m)(\varphi(lm)) \geq \varphi(l); \forall\, l, m \in S$.
Here φ is a LRI of H, hence $\varphi(0) \geq \varphi(l), \forall\, l \in H$.
(b) An LRh-ideal φ of H is stated to be a LRI φ of H, if for all $x, y, l, n \in S$
$l \oplus x \oplus n = y \oplus n \Rightarrow \varphi(l) \geq \inf\{\varphi(x), \varphi(y)\}$. Therefore φ is called both LRh-ideals of S.

Definition 2.4. (a) A tri-valued fuzzy set L of S is said to be tri-valued fuzzy LRI of H if the following properties are satisfied $\varphi(l \oplus m) \geq \inf\{\varphi(l), \varphi(m)\}, \quad \theta(l \oplus m) \leq \sup\{\theta(l), \theta(m)\}$ and $\varphi(lm) \geq \varphi(m)(\varphi(lm)) \geq \varphi(l); \theta(lm) \leq \theta(m)(\theta(lm)) \leq \theta(l) \forall\, l, m \in H$.
Clearly, if φ is a tri-valued LRFI (Left-right fuzzy ideal) of H, therefore $\varphi(0) \geq \varphi(l)$ and $\theta(0) \leq \theta(l), \forall\, l \in H$.
(b) A tri-valued LRFI L of H, is stated as tri-valued LRF ideals L of H, if for all $x, y, l, n \in S$
$l \oplus x \oplus n = y \oplus n \Rightarrow \varphi(l) \geq \inf\{\varphi(x), \varphi(y)\}$ and $\theta(l) \leq \sup\{\theta(x), \theta(y)\}$.
A tri-valued fuzzy set L is said to be an LRFI of H, if it both a tri-valued LRFI of H.

Definition 2.5. Let C be common universal, $\mathbb{F}$ the set of all factors, and L a trivalued fuzzy set over with the membership mapping $\varphi_L : \mathbb{F} \to Int([0.1])$, and non membership mapping $\theta_L : \mathbb{F} \to Int([0.1])\chi_L$ be an Tri-Value fuzzy set over C for every $l \in \mathbb{F}$. Then, $\lambda-$set, λ_L over Tri-value fuzzy set over C(Represented by TVF(C)) is stated by a function $\lambda_L(l)$ representing function

$$\lambda_L : \mathbb{F} \to TVF(C) \quad such\,that\; \lambda_L(l) = \emptyset,\; if\;\, l \in L \tag{2.5}$$

where λ_L is said to be tri-valued fuzzy rough calculation of $\lambda-$set,$\lambda_L(l)$ is a tri-valued fuzzy set said to be l-member of the $\lambda-$set, for every $l \in \mathbb{F}$. Thus an $\lambda-$set, λ_L over C can be stated by

$$\lambda_L = \{(l, (\varphi_L(l), \theta_L(l)), \eta_L(l)) : l \in \mathbb{F}, \eta_L(l) \in TVF(C)\} \tag{2.6}$$

Clearly, $\varphi_L(l) = 0$, $\theta_L(l) = 1$ and $\eta_L(l) = empty$, we don't show such element in the set. It necessities that the sets of every $\lambda-$sets over TVF(C) will be symbolized by $\lambda(C)$.

Definition 2.6. Let $\lambda_L \in \lambda(C)$.

1. If $\lambda_L(l) = \emptyset, \forall\, l \in \mathbb{F}$, then λ_L is said to be an $L-$empty, $\lambda-$set, and symbolized by $\lambda_{\emptyset_L}$.
2. If $L = \emptyset$, then the $L-$empty $\lambda-$set ($\lambda_{\emptyset_L}$) is symbolized by $\lambda_\emptyset$, where $\emptyset$ this means Tri-Value fuzzy empty set.

3. IF $\varphi_L(l) = 1$, $\theta_L(l) = 0$ and $\eta_L(l) = C$, $\forall\, l \in L$,, then λ_L is said to be $L-$common universal $\lambda-$set , symbolized by $\lambda_{\tilde{L}}$.

4. If L is equal to a tri-valued fuzzy common universal set over $\mathbb{F}$, then the $L-$common universal $\lambda-$set, is said to be common universal $\lambda-$set, symbolized by $\lambda_{\tilde{\mathbb{F}}}$. Here C means that tri-valued fuzzy common universal set.

Definition 2.7. Let $\lambda_L, \lambda_M \in \lambda(C)$. Then

1. λ_L be an TVF factor subset of λ_M, denoted by $\lambda_L \sqsubseteq \lambda_M$, if $\varphi_L(l) \leq \varphi_M(l)$, $\theta_L(l) \geq \theta_M(l)$ and $\eta_L(l) \sqsubseteq \eta_M(l)$.

2. λ_L and λ_M be an TVF factor equal, denoted by $\lambda_L = \lambda_M$, if $\varphi_L(l) = \varphi_M(l)$, $\theta_L(l) = \theta_M(l)$ and $K_L(l) = K_M(l)$, $\forall\, l \in \mathbb{F}$.

Definition 2.8. Let $\lambda_L \in \lambda(C)$. Then the compliment of λ_L, symbolized by λ'_L, is a TVF-soft fuzzy set stated by ($'$ meant compliment)

$$\lambda'_L(l) = 1 - \varphi(l),\ \theta'_L(l) = 1 - \theta(l)$$
$$\eta'_L(l) = C/\eta_L(l) \tag{2.7}$$

Definition 2.9. Let $\lambda_L, \lambda_M \in \lambda(C)$.

(a) The intersection of λ_L and λ_M, symbolized by $\lambda_L \sqcap \lambda_M$, is stated by

$$\lambda_L \sqcap \lambda_M = \{(l, (\varphi_{\lambda_L \sqcap \lambda_M}(l), \theta_{\lambda_L \sqcup \lambda_M}(l)), \eta_{\lambda_L \sqcap \lambda_M}(l)) : \in \mathbb{F}\} \tag{2.8}$$

where, $\varphi_{\lambda_L \sqcap \lambda_M}(l) = \inf\{\varphi_L(l), \varphi_M(l)\}$, $\quad \theta_{\lambda_L \sqcup \lambda_M}(l) = \sup\{\theta_L(l), \theta_M(l)\}$,
$\eta_{\lambda_L \sqcap \lambda_M}(l) = \{(r, \inf\{\varphi_{\eta_L(l)}(r), \varphi_{\eta_M(l)}(r)\}, \sup\{\theta_{\eta_L(l)}(r), \theta_{\eta_M(l)}(r)\}) : r \in C\}$

(b) The union of λ_L and λ_M, symbolized by $\lambda_L \sqcup \lambda_M$, is stated by

$$\lambda_L \sqcup \lambda_M = \{(l, (\varphi_{\lambda_L \sqcup \lambda_M}(l), \theta_{\lambda_L \sqcap \lambda_M}(l)), \eta_{\lambda_L \sqcup \lambda_M}(l)) : \in \mathbb{F}\} \tag{2.9}$$

where, $\varphi_{\lambda_L \sqcup \lambda_M}(l) = \sup\{\varphi_L(l), \varphi_M(l)\}$, $\quad \theta_{\lambda_L \sqcap \lambda_M}(l) = \inf\{\theta_L(l), \theta_M(l)\}$,
$\eta_{\lambda_L \sqcup \lambda_M}(l) = \{(r, \sup\{\varphi_{\eta_L(l)}(r), \varphi_{\eta_M(l)}(r)\}, \inf\{\theta_{\eta_L(l)}(r), \theta_{\eta_M(l)}(r)\}) : r \in C\}$
Here, $\eta_{L \sqcup M} = \eta_L(l)\tilde{\coprod}\eta_M(l)$, $\forall l \in \mathbb{F}$.

(c) The multiplication operator minimum ($\wedge$) of λ_L and λ_M, represented by $\lambda_L \wedge \lambda_M$, is stated as

$$\lambda_L \wedge \lambda_M = \{(l, (\varphi_{L\wedge M}(l,m), \theta_{L\wedge M}(l,m)), \eta_{L\wedge M}(l,m)) : \in \mathbb{F}\} \tag{2.10}$$

where, $\varphi_{L\wedge M}(l,m) = \inf\{\varphi_L(l), \varphi_M(m)\}$, $\quad \theta_{L\wedge M}(l,m) = \sup\{\theta_L(l), \theta_M(m)\}$,
$\eta_{L\wedge M}(l,m) = \{(v, \inf\{\varphi_{\eta_L(l)}(v), \varphi_{\eta_M(l)}(v)\}, \sup\{\theta_{\eta_L(l)}(v), \theta_{\eta_M(l)}(v)\}) : v \in C\}$
Here, $\eta_{L\wedge M} = \eta_L(l) \wedge \eta_M(l)$, $\forall l \in \mathbb{F}$.

3 Tri-valued soft fuzzy left right ideals (TVSFLRI)

Definition 3.1. Take H is a hemi-ring, $\mathbb{F}$ is a factors and L a tri-valued fuzzy set along with $\mathbb{F}$
$\lambda_L = \{(l, \varphi_L(l), \theta_L(l), \eta_L(l)) : l \in C,\ \eta_L(l) \in TVF(C)\} \in \lambda(C)$
Hence λ_L is called tri-valued soft fuzzy leftright ideals (TVSFLRI) over H, and if for any $l \in \mathbb{F}$, $\eta_L(l)$ be a tri-value fuzzy LRI of H.

Example 3.1. Take $H = \mathbb{Q}_3 = \{\frac{0}{10}, \frac{10}{10}, \frac{2}{10}\}$ is a hemi-ring, and $\mathbb{F} = \{x, y\}$ is a factor set. If $L = \left\{ \left(x, \left(\frac{3}{10}, \frac{6}{10}\right)\right), \left(y, \left(\frac{5}{10}, \frac{2}{10}\right)\right) \right\}$, $\eta_L(x) = \left\{ \left(\frac{0}{10}, \left(\frac{7}{10}, \frac{2}{10}\right)\right), \left(\frac{10}{10}, \left(\frac{6}{10}, \frac{3}{10}\right)\right), \left(\frac{20}{10}, \left(\frac{6}{10}, \frac{3}{10}\right)\right) \right\}$
$\eta_M(y) = \left\{ \left(\frac{0}{10}, \left(\frac{5}{10}, \frac{1}{10}\right)\right), \left(\frac{10}{10}, \left(\frac{3}{10}, \frac{6}{10}\right)\right), \left(\frac{20}{10}, \left(\frac{3}{10}, \frac{6}{10}\right)\right) \right\}$. Hence, λ_L be a TVF soft leftright ideals over H.

Example 3.2. Let $H = [a_{ij}]_n$ be a hemiring matrix, let $\mathbb{F} = \{x, y\}$ is a factor set if L is a tri-value fuzzy set over H is stated by

$$\varphi_L(l) = \begin{cases} \frac{2}{10}, & \text{if } l = m \\ \frac{6}{10}, & \text{if } l = y. \end{cases} \tag{3.1}$$

where $\theta_L(l) = 1 - \varphi_L(l)$. η_L is stated by

$$\varphi_{\eta_L(w)}(u) = \begin{cases} 0, & \text{if } u \textit{ is non triangular matrix} \\ 1, & \text{if } u \textit{ is triangular matrix.} \end{cases}$$
$$\theta_{\eta_L(w)}(u) = 1 - \varphi_{\eta_L(w)}(u)$$
$$\varphi_{\eta_L(q)}(v) = \begin{cases} 0, & \text{if } v \textit{ is non triangular matrix} \\ 1, & \text{if } v \textit{ is triangular matrix.} \end{cases}$$
$$\theta_{\eta_L(q)}(v) = 1 - \varphi_{\eta_L(q)}(v) \tag{3.2}$$

For every $l \in \mathbb{F}$, if we confirm that $\eta_L(l)$ be an Tri-valued left-right fuzzy ideal of H and λ_L be an Tri-valued soft fuzzy ideal over H.

Theorem 3.1. *Let $\lambda_L, \lambda_M \in$ TVF(C) be LRI over H, hence "$\lambda_L \overline{\overline{\sqcap}} \lambda_M$" is an tri-valued soft fuzzy LRI over H.*

Proof. Here we represented $\lambda_L \overline{\overline{\sqcap}} \lambda_M = \lambda_{L \overline{\overline{\sqcap}} M}$. For every $l \in \mathbb{F}$, we have

$$\varphi_{L \overline{\overline{\sqcap}} M} = \inf\{\varphi_L(l), \varphi_M(l)\}$$
$$\varphi_{L \overline{\overline{\sqcap}} M} = \sup\{\varphi_L(l), \varphi_M(l)\}$$

Now we show that $\eta_L(l) \overline{\overline{\sqcup}} \eta_M(l)$ be an LRI of H. For every $l \in \mathbb{F}$, $h_1, h_2 \in H$

$$\begin{aligned}
\left(\varphi_{\eta_L(l)} \overline{\overline{\sqcap}} \varphi_{\eta_M(l)}\right)(h_1 + h_2) &= \inf\{\varphi_{\eta_L(l)}(h_1 + h_2), \varphi_{\eta_M(l)}(h_1 + h_2)\} \\
&\geq \inf\{\inf\{\varphi_{\eta_L(l)}(h_1), \varphi_{\eta_L(l)}(h_2)\}, \inf\{\varphi_{\eta_M(l)}(h_1), \varphi_{\eta_M(l)}(h_2)\}\} \\
&= \inf\{\inf\{\varphi_{\eta_L(l)}(h_1), \varphi_{\eta_M(l)}(h_1)\}, \inf\{\varphi_{\eta_L(l)}(h_2), \varphi_{\eta_M(l)}(h_2)\}\} \\
&= \inf\left\{ \left(\varphi_{\eta_L(l)} \overline{\overline{\sqcap}} \varphi_{\eta_M(l)}\right)(h_1), \left(\varphi_{\eta_L(l)} \overline{\overline{\sqcap}} \varphi_{\eta_M(l)}\right)(h_2)\right\} \qquad (3.3) \\
\left(\varphi_{\eta_L(l)} \overline{\overline{\sqcup}} \varphi_{\eta_M(l)}\right)(h_1 + h_2) &= \sup\{\varphi_{\eta_L(l)}(h_1 + h_2), \varphi_{\eta_M(l)}(h_1 + h_2)\} \\
&\geq \sup\{\inf\{\varphi_{\eta_L(l)}(h_1), \varphi_{\eta_L(l)}(h_2)\}, \sup\{\varphi_{\eta_M(l)}(h_1), \varphi_{\eta_M(l)}(h_2)\}\} \\
&= \sup\{\sup\{\varphi_{\eta_L(l)}(h_1), \varphi_{\eta_M(l)}(h_1)\}, \sup\{\varphi_{\eta_L(l)}(h_2), \varphi_{\eta_M(l)}(h_2)\}\} \\
&= \sup\left\{ \left(\varphi_{\eta_L(l)} \overline{\overline{\sqcup}} \varphi_{\eta_M(l)}\right)(h_1), \left(\varphi_{\eta_L(l)} \overline{\overline{\sqcup}} \varphi_{\eta_M(l)}\right)(h_2)\right\} \qquad (3.4)
\end{aligned}$$

Hence $\varphi_{\eta_L(l)}(h_1h_2) \geq \varphi_{\eta_L(l)}(h_2)$, $\quad \theta_{\eta_L(l)}(h_1h_2) \leq \theta_{\eta_L(l)}(h_2)$, also $\varphi_{\eta_M(l)}(h_1h_2) \geq \varphi_{\eta_M(l)}(h_2)$, $\quad \theta_{\eta_M(l)}(h_1h_2) \leq \theta_{\eta_M(l)}(h_2)$, hence

$$\begin{aligned}
\left(\varphi_{\eta_L(l)} \overline{\overline{\sqcap}} \varphi_{\eta_M(l)}\right)(h_1h_2) &= \inf\{\varphi_{\eta_L(l)}(h_1h_2), \varphi_{\eta_M(l)}(h_1h_2)\} \\
&\geq \inf\{\varphi_{\eta_L(l)}(s_2), \varphi_{\eta_M(l)}(h_2)\} \\
&= \left(\varphi_{\eta_L(l)} \overline{\overline{\sqcap}} \varphi_{\eta_M(l)}\right)(h_2) \qquad (3.5) \\
\left(\theta_{\eta_L(l)} \overline{\overline{\sqcap}} \theta_{\eta_M(l)}\right)(h_1h_2) &= \inf\{\theta_{\eta_L(l)}(h_1h_2), \theta_{\eta_M(l)}(h_1h_2)\} \\
&\geq \inf\{\theta_{\eta_L(l)}(s_2), \theta_{\eta_M(l)}(h_2)\} \\
&= \left(\theta_{\eta_L(l)} \overline{\overline{\sqcap}} \theta_{\eta_M(l)}\right)(h_2) \qquad (3.6)
\end{aligned}$$

Therefore we look $\eta_L(l)\overline{\overline{\sqcup}}\eta_M(l)$ be an tri-valued left fuzzy ideal of H. In the same way we show that the proof of right ideal of H. For every $l \in \mathbb{F}$, $x, y, h, n \in H$, let $h \oplus x \oplus n = y \oplus n$, then

$$\begin{aligned}
\left(\varphi_{\eta_L(l)}\overline{\overline{\sqcap}}\varphi_{\eta_M(l)}\right)(h) &= \inf\{\varphi_{\eta_L(l)}(h), \varphi_{\eta_M(l)}(h)\} \\
&\geq \inf\{\inf\{\varphi_{\eta_L(l)}(x), \varphi_{\eta_L(l)}(y)\}, \inf\{\varphi_{\eta_M(l)}(x), \varphi_{\eta_M(l)}(y)\}\} \\
&= \inf\{\inf\{\varphi_{\eta_L(l)}(x), \varphi_{\eta_M(l)}(x)\}, \inf\{\varphi_{\eta_L(l)}(y), \varphi_{\eta_M(l)}(y)\}\} \\
&= \inf\left\{\left(\varphi_{\eta_L(l)}\overline{\overline{\sqcap}}\varphi_{\eta_M(l)}\right)(x), \left(\varphi_{\eta_L(l)}\overline{\overline{\sqcap}}\varphi_{\eta_M(l)}\right)(y)\right\} \qquad (3.7)\\
\left(\theta_{\eta_L(l)}\overline{\overline{\sqcap}}\theta_{\eta_M(l)}\right)(h) &= \inf\{\theta_{\eta_L(l)}(h), \theta_{\eta_M(l)}(h)\} \\
&\geq \sup\{\sup\{\theta_{\eta_L(l)}(x), \theta_{\eta_L(l)}(y)\}, \sup\{\theta_{\eta_M(l)}(x), \theta_{\eta_M(l)}(y)\}\} \\
&= \sup\{\sup\{\theta_{\eta_L(l)}(x), \theta_{\eta_M(l)}(x)\}, \sup\{\theta_{\eta_L(l)}(y), \theta_{\eta_M(l)}(y)\}\} \\
&= \sup\left\{\left(\theta_{\eta_L(l)}\overline{\overline{\sqcap}}\theta_{\eta_M(l)}\right)(x), \left(\theta_{\eta_L(l)}\overline{\overline{\sqcap}}\theta_{\eta_M(l)}\right)(y)\right\} \qquad (3.8)
\end{aligned}$$

Therefore we look $\eta_L(l)\overline{\overline{\sqcup}}\eta_M(l)$ be an tri-valued left right fuzzy ideal of H.
ie., $\varphi_{L\overline{\overline{\sqcap}}M}(l) = \eta_L(l)\overline{\overline{\sqcup}}\eta_M(l)$ be an tri-valued left right fuzzy ideal of H.
We identify that the "$\overline{\overline{\sqcap}}$" of all Tri-valued left-right fuzzy ideals over H. Hence we assume the "$\overline{\overline{\sqcup}}$" of Tri-value LRFI over H is also an Tri-value LRI over H. □

Notation: Take $\lambda_L, \lambda_M \in$ TVF(C) be soft LRFI over H. If ordered sequence for every $l \in \mathbb{F}$, $h_1, h_2 \in H$,

(a) $\varphi_{\eta_L(h_1)} \geq \varphi_{\eta_L(h_2)} \Rightarrow \varphi_{\eta_M(h_1)} \geq \varphi_{\eta_M(h_2)}$

(b) $\theta_{\eta_L(h_1)} \geq \theta_{\eta_L(h_2)} \Rightarrow \theta_{\eta_M(h_1)} \geq \theta_{\eta_M(h_2)}$

Theorem 3.2. *Take $\lambda_L, \lambda_M \in$ TVF(C) be the tri-valued left right soft fuzzy ideals(LRSFI) over H, along with sequence order values $\lambda_L\overline{\overline{\sqcup}}\lambda_M$ is still an Tri-value soft left-right ideals over H.*

Proof. The proof is straightforward in Theorem 3.1. □

Theorem 3.3. *Take $\lambda_L, \lambda_M \in$ TVF(C) be the TVFLRI over H. Therefore $\lambda_L \wedge \lambda_M$ be an Tri-value left-right fuzzy soft ideals $H \otimes H$.*

Proof. The proof is straightforward in Theorem 3.2. □

Definition 3.2. Take λ_L is a TVFLRI over H and λ_M is a TVFSS subset over H. Then λ_L is called TVFLRI of λ_M, if λ_L be an TVFSS subset of λ_M.

Example 3.3. Take $H = \mathbb{Q}_4 = \{0, \frac{10}{10}, \frac{20}{10}, \frac{30}{10}\}$ is a hemiring, and $\mathbb{F} = \{x, y\}$ is a factor set. If
$L = \left\{ \left(x, \left(\frac{1}{10}, \frac{7}{10}\right)\right), \left(y, \left(\frac{3}{10}, \frac{6}{10}\right)\right) \right\}$
$\eta_L(x) = \left\{ \left(0, \left(\frac{4}{10}, \frac{5}{10}\right)\right), \left(\frac{10}{10}, \left(\frac{2}{10}, \frac{7}{10}\right)\right), \left(\frac{20}{10}, \left(\frac{3}{10}, \frac{6}{10}\right)\right), \left(\frac{30}{10}, \left(\frac{2}{10}, \frac{6}{10}\right)\right) \right\}$
$\eta_L(y) = \left\{ \left(0, \left(\frac{6}{10}, \frac{3}{10}\right)\right), \left(\frac{10}{10}, \left(\frac{1}{10}, \frac{6}{10}\right)\right), \left(\frac{20}{10}, \left(\frac{3}{10}, \frac{5}{10}\right)\right), \left(\frac{30}{10}, \left(\frac{2}{10}, \frac{7}{10}\right)\right) \right\}$
$M = \left\{ \left(x, \left(\frac{5}{10}, \frac{3}{10}\right)\right), \left(y, \left(\frac{6}{10}, \frac{3}{10}\right)\right) \right\}$
$\eta_M(x) = \left\{ \left(0, \left(\frac{5}{10}, \frac{2}{10}\right)\right), \left(\frac{10}{10}, \left(\frac{3}{10}, \frac{6}{10}\right)\right), \left(\frac{20}{10}, \left(\frac{4}{10}, \frac{4}{10}\right)\right), \left(\frac{30}{10}, \left(\frac{2}{10}, \frac{6}{10}\right)\right) \right\}$
$\eta_M(y) = \left\{ \left(0, \left(\frac{9}{10}, \frac{0}{10}\right)\right), \left(\frac{10}{10}, \left(\frac{8}{10}, \frac{2}{10}\right)\right), \left(\frac{20}{10}, \left(\frac{9}{10}, \frac{1}{10}\right)\right), \left(\frac{30}{10}, \left(\frac{7}{10}, \frac{3}{10}\right)\right) \right\}$

Therefore, λ_L be an TVFLRI over H, λ_M be an TVFLRI over λ_M.

4 Tri-value equivalent soft fuzzy left right ideals (TVESFLRI)

Definition 4.1. Let $\lambda_L = \{(l, \varphi_L(l), \theta_L(l), \eta_L(l)) : l \in A,\, \eta_L(l) \in TVF(H)\}$ and let $\lambda_M = \{(m, \varphi_M(m), \theta_M(m), \eta_M(m)) : m \in B,\, \eta_M(m) \in TVF(P)\}$ be an Tri-value soft fuzzy LRI over hemirings H and P in that order. If $g : H \to P$ and $f : A \to B$ are two mapping , then (g, f) is said to be an Tri-value soft fuzzy morphism such that (g, f) is an Tri-value soft fuzzy morphism over left right ideals from λ_L to λ_M. Hence we written by $\lambda_L \equiv \lambda_M$, if the below property fulfilled:
(i) g is an epimorphism from H to P.
(ii) f is a one-one onto mapping.
(iii) $\varphi_L(l) = \varphi_M(f(l))$, $\theta_L(l) = \theta_M(f(l))$ and $g(\eta_L(l)) = \eta_M(f(l))$, $\forall\, l \in A$.

Example 4.1. Let $H = (\mathbb{Q}, \oplus, \otimes)$ and $P = (2\mathbb{Q}, \oplus, \otimes)$, $A = \left(\frac{10}{10}, \frac{20}{10}\right)$ and $B = \left(\frac{30}{10}, \frac{60}{10}\right)$. State a homomorphism g from H into P by $f(h) = 2h$, for $e \in H$ and a function f from A into B by $f(l) = 3l$, $\forall\, l \in A$. Let L be an tri-value fuzzy set over A stated by

$$L = \left\{ \left(\frac{10}{10}, \left(\frac{3}{10}, \frac{6}{10}\right)\right), \left(\frac{20}{10} \left(\frac{7}{10}, \frac{2}{10}\right)\right) \right\}$$

and M be an tri-value fuzzy set over B stated by

$$M = \left\{ \left(\frac{30}{10}, \left(\frac{3}{10}, \frac{6}{10}\right)\right), \left(\frac{60}{10} \left(\frac{7}{10}, \frac{2}{10}\right)\right) \right\}$$

. Let $\eta_L : A \to \mathbb{F}(H)$ stated by

$$\left(\eta_L\left(\frac{10}{10}\right)\right)(h) = \begin{cases} \left(\frac{2}{10}, \frac{7}{10}\right), & h = \frac{20}{10}p + 1,\, p \in \mathbb{Q} \\ \left(\frac{6}{10}, \frac{3}{10}\right), & h = \frac{20}{10}p,\, p \in \mathbb{Q} \end{cases}$$
$$\left(\eta_L\left(\frac{20}{10}\right)\right)(h) = \begin{cases} \left(\frac{3}{10}, \frac{6}{10}\right), & h = \frac{20}{10}p + 1,\, p \in \mathbb{Q} \\ \left(\frac{4}{10}, \frac{5}{10}\right), & h = \frac{20}{10}p,\, p \in \mathbb{Q} \end{cases} \tag{4.1}$$

Let $\eta_M : B \to \mathbb{F}(P)$ stated by

$$\left(\eta_M\left(\frac{40}{10}\right)\right)(h) = \begin{cases} \left(\frac{2}{10}, \frac{7}{10}\right), & h = \frac{40}{10}p + 1,\, p \in \mathbb{Q} \\ \left(\frac{6}{10}, \frac{3}{10}\right), & h = \frac{40}{10}p,\, p \in \mathbb{Q} \end{cases}$$
$$\left(\eta_M\left(\frac{80}{10}\right)\right)(h) = \begin{cases} \left(\frac{3}{10}, \frac{6}{10}\right), & h = \frac{40}{10}p + 1,\, p \in \mathbb{Q} \\ \left(\frac{4}{10}, \frac{5}{10}\right), & h = \frac{40}{10}p,\, p \in \mathbb{Q} \end{cases} \tag{4.2}$$

Note that λ_L and λ_M are tri-value soft fuzzy left-right idelas over H and P, in that order, we can quickly see that $\varphi_L(l) = \varphi_M(f(l))$ and we can infer that $g(\varphi_L(l)) = \varphi_M(f(l))$, Hence (g, f) is a tri-value soft fuzzy homomorphism from λ_L to λ_M.

Lemma 4.1. *If $g : H \to P$ epimorphism of hemirings and L is a tri-value fuzzy LRI of H, then $g(L)$ is an tri-vlaue fuzzy ideal of P.*

Theorem 4.2. *Let $\lambda_L = \{(l, \varphi_L(l), \theta_L(l), \eta_L(l)) : l \in A,\, \eta_L(l) \in TVF(H)\}$ tri-value soft fuzzy LRI over H, and $\lambda_M = \{(m, \varphi_M(m), \theta_M(m), \eta_M(m)) : m \in B,\, \eta_M(m) \in TVF(P)\}$ be a tri-value soft fuzzy LRI over P. If λ_L is a tri-value soft fuzzy homomorphic to λ_M, then λ_M is a tri-value soft fuzzy left right ideal over P.*

Proof. Let (g, f) be a tri-value soft fuzzy homomorphism from λ_L to λ_M. Since λ_L is an Tri-value soft fuzzy left right ideal over H., $g(H) = P$ and $\lambda_L(l)$ is a fuzzy LRI of H, $\forall\, l \in A$. Now, $\forall\, m \in B$, $\exists\, l \in A$ such that $f(l) = m$, therefore $\varphi_M(m) = \varphi_L(f(l)) = \varphi_L(l)$, $\theta_M(m) = \theta_L(f(l)) = \theta_L(l)$.
By the above lemma(4.1) $\eta_M(m) = \eta_L(f(l)) = g(\eta_L(l))$ is a tri-value fuzzy LRI of their hemiring P. So φ_M is a tri-vlaue soft fuzzy LRI over P as well. □

Definition 4.2. Let $\lambda_L = \{(l, \varphi_L(l), \theta_L(l), \eta_L(l)) : l \in A,\ \eta_L(l) \in TVF(H)\}$ be a tri-value soft fuzzy left-rights over hemiring H. Then λ_L is called a tri-value equivalent soft fuzzy LRI over H, if for every $\varphi_L(l) = \varphi_m(l)$ and $\theta_L(l) = \theta_m(l)$, we have $\eta_L(l) = \varphi_L(m)$.
By Definitions 4.1 and 2.5 we can simply find that λ'_L ($'$ means complement) is a tri-value equivalent tri-value soft fuzzy left-right ideals over H if λ_L is a tri-value equivalent soft fuzzy left-right ideal over H.

Example 4.2. Assume that

$$H_{lm} = [a_{ij}]_{2\times 2} = \left\{ \begin{bmatrix} l & m \\ l & m \end{bmatrix} / l, m \in \mathbb{Q} = \{0, 1\} \right\} \tag{4.3}$$

is a hemiring, $\mathbb{F} = \{l_1, l_2, l_3, l_4\}$ be the factors set, and L is an Tri-vlaue fuzzy set over H stated by $L = \left\{ \left(l_1, \left(\frac{3}{10}, \frac{7}{10}\right)\right), \left(l_2, \left(\frac{6}{10}, \frac{2}{10}\right)\right), \left(l_3, \left(\frac{6}{10}, \frac{2}{10}\right)\right), \left(l_4, \left(\frac{7}{10}, \frac{1}{10}\right)\right) \right\}$

$$\eta_L(l_1) = \left\{ \left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \left(\tfrac{7}{10}, \tfrac{1}{10}\right) \right), \left(\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \left(\tfrac{3}{10}, \tfrac{6}{10}\right) \right), \left(\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \left(\tfrac{3}{10}, \tfrac{6}{10}\right) \right), \left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \left(\tfrac{6}{10}, \tfrac{2}{10}\right) \right) \right\}$$
$$\eta_L(l_2) = \left\{ \left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \left(\tfrac{5}{10}, \tfrac{1}{10}\right) \right), \left(\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \left(\tfrac{2}{10}, \tfrac{8}{10}\right) \right), \left(\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \left(\tfrac{2}{10}, \tfrac{8}{10}\right) \right), \left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \left(\tfrac{3}{10}, \tfrac{6}{10}\right) \right) \right\}$$
$$\eta_L(l_3) = \left\{ \left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \left(\tfrac{5}{10}, \tfrac{1}{10}\right) \right), \left(\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \left(\tfrac{2}{10}, \tfrac{8}{10}\right) \right), \left(\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \left(\tfrac{2}{10}, \tfrac{8}{10}\right) \right), \left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \left(\tfrac{3}{10}, \tfrac{6}{10}\right) \right) \right\}$$
$$\eta_L(l_4) = \left\{ \left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \left(\tfrac{8}{10}, \tfrac{1}{10}\right) \right), \left(\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \left(\tfrac{5}{10}, \tfrac{4}{10}\right) \right), \left(\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \left(\tfrac{5}{10}, \tfrac{4}{10}\right) \right), \left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \left(\tfrac{7}{10}, \tfrac{3}{10}\right) \right) \right\}$$

Hence η_L is a tri-value equivalent soft LRI over H_{lm}.

Notation: Let $\lambda_L = \{(l, \varphi_L(l), \theta_L(l), \eta_L(l)) : l \in \mathbb{F},\ \eta_L(l) \in TVF(H)\}$ and $\lambda_M = \{(m, \varphi_M(m), \theta_M(m), \eta_M(m)) : m \in \mathbb{F},\ \eta_M(m) \in TVF(H)\}$ are tri-value equivalent soft fuzzy LRI over hemiring H, then $\lambda_L \overline{\overline{\sqcup}} \lambda_M$ are always not tri-value soft fuzzy LRI over H.

Example 4.3. Assume that $H = \mathbb{Q}_3$, $\mathbb{F}\{l_1, l_2, l_3\}$. Let λ_L be an Tri-value soft fuzzy left right ideals over H stated by $A = \left\{ \left(l_1, \left(\frac{3}{10}, \frac{5}{10}\right)\right), \left(l_2, \left(\frac{3}{10}, \frac{5}{10}\right)\right), \left(l_3, \left(\frac{2}{10}, \frac{6}{10}\right)\right) \right\}$
$\eta_L(l_1) = \left\{ \left(\frac{0}{10}, \left(\frac{5}{10}, \frac{4}{10}\right)\right), \left(\frac{10}{10}, \left(\frac{3}{10}, \frac{6}{10}\right)\right), \left(\frac{3}{10}, \frac{6}{10}\right) \right\}$
$\eta_L(l_2) = \left\{ \left(\frac{0}{10}, \left(\frac{5}{10}, \frac{4}{10}\right)\right), \left(\frac{10}{10}, \left(\frac{3}{10}, \frac{6}{10}\right)\right), \left(\frac{3}{10}, \frac{6}{10}\right) \right\}$
$\eta_L(l_3) = \left\{ \left(\frac{0}{10}, \left(\frac{6}{10}, \frac{3}{10}\right)\right), \left(\frac{10}{10}, \left(\frac{4}{10}, \frac{5}{10}\right)\right), \left(\frac{4}{10}, \frac{5}{10}\right) \right\}$

Let λ_M be an Tri-value soft fuzzy left right ideals over H stated by

$$\begin{aligned}
A &= \left\{ \left(m_1, \left(\frac{2}{10}, \frac{6}{10}\right)\right), \left(m_2, \left(\frac{6}{10}, \frac{3}{10}\right)\right), \left(m_3, \left(\frac{6}{10}, \frac{3}{10}\right)\right) \right\} \\
\eta_M(m_1) &= \left\{ \left(\frac{0}{10}, \left(\frac{8}{10}, \frac{1}{10}\right)\right), \left(\frac{10}{10}, \left(\frac{1}{10}, \frac{7}{10}\right)\right), \left(\frac{1}{10}, \frac{7}{10}\right) \right\} \\
\eta_M(m_2) &= \left\{ \left(\frac{0}{10}, \left(\frac{6}{10}, \frac{3}{10}\right)\right), \left(\frac{10}{10}, \left(\frac{2}{10}, \frac{5}{10}\right)\right), \left(\frac{2}{10}, \frac{5}{10}\right) \right\} \\
\eta_M(m_3) &= \left\{ \left(\frac{0}{10}, \left(\frac{6}{10}, \frac{3}{10}\right)\right), \left(\frac{10}{10}, \left(\frac{2}{10}, \frac{5}{10}\right)\right), \left(\frac{2}{10}, \frac{5}{10}\right) \right\} \qquad (4.4)
\end{aligned}$$

It is note that λ_L and λ_M are tri-value equivalent soft fuzzy LRI over H. We can look that

$$\begin{aligned}
A\bar{\bar{\sqcap}}B &= \left\{ \left(l_1, \left(\frac{2}{10}, \frac{6}{10}\right)\right), \left(l_2, \left(\frac{3}{10}, \frac{5}{10}\right)\right), \left(l_3, \left(\frac{2}{10}, \frac{6}{10}\right)\right) \right\} \\
\eta_{(A\bar{\bar{\sqcap}}B)}(l_1) &= \eta_{(A\bar{\bar{\sqcap}}B)}(l_3) \qquad (4.5) \\
A\bar{\bar{\sqcup}}B &= \left\{ \left(l_1, \left(\frac{3}{10}, \frac{5}{10}\right)\right), \left(l_2, \left(\frac{6}{10}, \frac{3}{10}\right)\right), \left(l_3, \left(\frac{6}{10}, \frac{3}{10}\right)\right) \right\} \\
\eta_{(A\bar{\bar{\sqcup}}B)}(l_2) &= \eta_{(A\bar{\bar{\sqcup}}B)}(l_3) \qquad (4.6) \\
But, \quad \eta_{(A\bar{\bar{\sqcap}}B)}(l_1) &= \left\{ \left(\frac{0}{10}, \left(\frac{5}{10}, \frac{4}{10}\right)\right), \left(\frac{10}{10} \left(\frac{1}{10}, \frac{7}{10}\right)\right), \left(\frac{1}{10}, \frac{7}{10}\right) \right\} \\
\eta_{(A\bar{\bar{\sqcap}}B)}(l_3) &= \left\{ \left(\frac{0}{10}, \left(\frac{6}{10}, \frac{3}{10}\right)\right), \left(\frac{10}{10} \left(\frac{2}{10}, \frac{5}{10}\right)\right), \left(\frac{2}{10}, \frac{5}{10}\right) \right\} \\
\eta_{(A\bar{\bar{\sqcup}}B)}(l_2) &= \left\{ \left(\frac{0}{10}, \left(\frac{6}{10}, \frac{3}{10}\right)\right), \left(\frac{10}{10} \left(\frac{3}{10}, \frac{5}{10}\right)\right), \left(\frac{3}{10}, \frac{5}{10}\right) \right\} \\
\eta_{(A\bar{\bar{\sqcup}}B)}(l_3) &= \left\{ \left(\frac{0}{10}, \left(\frac{6}{10}, \frac{3}{10}\right)\right), \left(\frac{10}{10} \left(\frac{4}{10}, \frac{5}{10}\right)\right), \left(\frac{4}{10}, \frac{5}{10}\right) \right\} \qquad (4.7)
\end{aligned}$$

Hence $\eta_{(A\bar{\bar{\sqcap}}B)}$ and $\eta_{(A\bar{\bar{\sqcup}}B)}$ are not tri-value equivalent soft fuzzy leftright ideals over H.

Theorem 4.3. *Let $\lambda_L = \{(l, \varphi_L(l), \theta_L(l), \eta_L(l)) : l \in \mathbb{F},\ \eta_L(l) \in TVF(H)\}$ be a tri-value soft fuzzy LRI over H, and $\lambda_M = \{(m, \varphi_M(m), \theta_M(m), \eta_M(m)) : m \in \mathbb{F},\ \eta_M(m) \in TVF(P)\}$ be a tri-value soft fuzzy LRI over P. If λ_L is a tri-value soft fuzzy homomorphic to λ_M, then λ_M is a tri-value soft fuzzy left right ideal over P.*

Proof. Let (g, f) be a tri-value soft fuzzy homomorphism from λ_L to λ_M. Since λ_L is a tri-value soft fuzzy left right ideal over H, we have $\eta_L(l_1) = \eta_L(l_2)$, if $\varphi_L(l_1) = \varphi_L(l_2)$, $\theta_L(l_1) = \theta_L(l_2)$ for any $l_1, l_2 \in A$.
Now for every $m_1, m_2 \in B$, then $\exists\, l_1, l_2 \in A$ with $f(l_1) = m_1$, $f(l_2) = m_2$. Since $\varphi_M(m_1) = \varphi_M(f(l_1)) = \varphi_M(m_2)$ and all over again $\theta_M(m_1) = \theta_M(m_2)$. And then $\eta_M(m_1) = \eta_M(f(l_1)) = g(\eta_L(l_1)) = g(\eta_L(l_2)) = \eta_M(f(l_2)) = \eta_M(l_2)$. Hence λ_M necessirly be a tri-value equivalent soft fuzzy LRI over P as well. □

Definition 4.3. Let $\lambda_L = \{(l, \varphi_L(l), \theta_L(l), \eta_L(l)) : l \in \mathbb{F},\ \eta_L(l) \in TVF(H)\}$ be an tri-value soft fuzzy LRI over H. Then λ_L is called an Tri-value non-decreasing soft fuzzy LRI over H, if for every $l, m \in \mathbb{F}$, $\varphi_L(l) \leq \varphi_L(m)$ and $\theta_L(l) \geq \theta_L(m)$, we have $\eta_L(l) \sqsubseteq \varphi_L(m)$.
And λ_L is called a tri-value non-increasing soft fuzzy LRI over H, if for every $l, m \in \mathbb{F}$, $\varphi_L(l) \leq \varphi_L(m)$ and $\theta_L(l) \leq \theta_L(m)$, we have $\eta_L(l) \sqsupseteq \varphi_L(m)$.

Example 4.4. Let $H = \mathbb{Q}_3$, $\mathbb{F}\{l_1, l_2, l_3\}$. Let L be a fuzzy set over $\mathbb{F}$ stated by $L = \left\{ \left(l_1, \left(\frac{1}{10}, \frac{8}{10}\right)\right), \left(l_2, \left(\frac{4}{10}, \frac{6}{10}\right)\right), \left(l_3, \left(\frac{7}{10}, \frac{3}{10}\right)\right) \right\}$

$\eta_L(l_1) = \left\{ \left(\frac{0}{10}, \left(\frac{2}{10}, \frac{6}{10}\right)\right), \left(\frac{10}{10}, \left(\frac{1}{10}, \frac{8}{10}\right)\right), \left(\frac{1}{10}, \frac{8}{10}\right) \right\}$

$\eta_L(l_2) = \left\{ \left(\frac{0}{10}, \left(\frac{6}{10}, \frac{4}{10}\right)\right), \left(\frac{10}{10}, \left(\frac{3}{10}, \frac{5}{10}\right)\right), \left(\frac{3}{10}, \frac{5}{10}\right) \right\}$

$\eta_L(l_3) = \left\{ \left(\frac{0}{10}, \left(\frac{9}{10}, \frac{0}{10}\right)\right), \left(\frac{10}{10}, \left(\frac{7}{10}, \frac{2}{10}\right)\right), \left(\frac{7}{10}, \frac{2}{10}\right) \right\}$

Hence η_L is an Tri-value non-decreasing soft fuzzy hemiring ideals over H.

Notation: Let $\lambda_L = \{(l, \varphi_L(l), \theta_L(l), \eta_L(l)) : l \in \mathbb{F}, \eta_L(l) \in TVF(H)\}$ and $\lambda_M = \{(m, \varphi_M(m), \theta_M(m), \eta_M(m)) : m \in \mathbb{F},\ \eta_M(m) \in TVF(H)\}$ be tri-value non-decreasing soft fuzzy left-right ideals over H. Then $\lambda_L \overline{\overline{\sqcup}} \lambda_M$ are always not a tri-value non-decreasing soft fuzzy LRI over H.

Example 4.5. Assume that $H = \mathbb{Q}_3$, $\mathbb{F}\{l_1, l_2\}$. Let L be a tri-value fuzzy set over $\mathbb{F}$ stated by
$L = \left\{ \left(l_1, \left(\frac{3}{10}, \frac{6}{10}\right)\right), \left(l_2, \left(\frac{5}{10}, \frac{2}{10}\right)\right) \right\}$

$\eta_L(l_1) = \left\{ \left(\frac{4}{10}, \frac{5}{10}\right), \left(\frac{1}{10}, \frac{6}{10}\right), \left(\frac{1}{10}, \frac{6}{10}\right) \right\}$

$\eta_L(l_2) = \left\{ \left(\frac{6}{10}, \frac{2}{10}\right), \left(\frac{5}{10}, \frac{3}{10}\right), \left(\frac{5}{10}, \frac{3}{10}\right) \right\}$
$M = \left\{ \left(l_1, \left(\frac{9}{10}, \frac{0}{10}\right)\right), \left(l_2, \left(\frac{4}{10}, \frac{3}{10}\right)\right) \right\}$
$\eta_M(l_1) = \left\{ \left(\frac{6}{10}, \frac{2}{10}\right), \left(\frac{4}{10}, \frac{5}{10}\right), \left(\frac{4}{10}, \frac{5}{10}\right) \right\}$
$\eta_M(l_2) = \left\{ \left(\frac{3}{10}, \frac{5}{10}\right), \left(\frac{2}{10}, \frac{8}{10}\right), \left(\frac{2}{10}, \frac{8}{10}\right) \right\}$

Note that, it is clearly λ_L and λ_Y are Tri-value soft fuzzy LRI over H. We can look that

$$L\overline{\overline{\sqcap}}M = \left\{ \left(l_1, \left(\frac{3}{10}, \frac{6}{10}\right)\right), \left(l_2, \left(\frac{4}{10}, \frac{3}{10}\right)\right) \right\}$$
$$L\overline{\overline{\sqcup}}M = \left\{ \left(l_1, \left(\frac{9}{10}, \frac{0}{10}\right)\right), \left(l_2, \left(\frac{5}{10}, \frac{2}{10}\right)\right) \right\} \tag{4.8}$$

$$But, \quad \eta_{(L\overline{\overline{\sqcap}}M)}(l_1) = \left\{ \left(\frac{4}{10}, \frac{5}{10}\right), \left(\frac{1}{10}, \frac{6}{10}\right), \left(\frac{1}{10}, \frac{6}{10}\right) \right\}$$
$$\eta_{(L\overline{\overline{\sqcap}}M)}(l_2) = \left\{ \left(\frac{3}{10}, \frac{5}{10}\right), \left(\frac{2}{10}, \frac{8}{10}\right), \left(\frac{2}{10}, \frac{8}{10}\right) \right\}$$
$$\eta_{(L\overline{\overline{\sqcup}}M)}(l_1) = \left\{ \left(\frac{6}{10}, \frac{2}{10}\right), \left(\frac{4}{10}, \frac{5}{10}\right), \left(\frac{4}{10}, \frac{5}{10}\right) \right\}$$
$$\eta_{(L\overline{\overline{\sqcup}}M)}(l_2) = \left\{ \left(\frac{6}{10}, \frac{2}{10}\right), \left(\frac{5}{10}, \frac{3}{10}\right), \left(\frac{5}{10}, \frac{3}{10}\right) \right\} \tag{4.9}$$

Then $\eta_L\tilde{\sqcap}\eta_M$ and $\eta_L\tilde{\sqcup}\eta_M$ are not tri-value soft fuzzy leftright ideals over H.

Theorem 4.4. *Let $\lambda_L = \{(l, \varphi_L(l), \theta_L(l), \eta_L(l)) : l \in \mathbb{F},\ \eta_L(l) \in TVF(H)\}$ be a tri-value soft fuzzy LRI over H, and $\lambda_M = \{(m, \varphi_M(m), \theta_M(m), \eta_M(m)) : m \in \mathbb{F},\ \eta_M(m) \in TVF(P)\}$ be a tri-value equivalent soft fuzzy set over hemiring P. If λ_L is a tri-value soft fuzzy homomorphic to λ_M, then λ_M is a tri-value non-decreasing soft fuzzy left right ideal over H.*

Proof. Let (g, f) be a tri-value soft fuzzy homomorphism from λ_L to λ_M, is an Tri-value soft fuzzy left right ideal over H, for all $l_1, l_2 \in A$, $\varphi_L(l_1) \leq \varphi_L(l_2)$ and $\theta_L(l_1) \geq \theta_L(l_2)$, we have $\eta_L(l_1) \sqsubseteq \eta_L(l_2)$.
Now for every $m_1, m_2 \in B$, then $\exists\, l_1, l_2 \in A$ with $f(l_1) = m_1,\ f(l_2) = m_2$. Since $\varphi_M(m_1) = \varphi_M(f(l_2)) = \varphi_M(l_2)$ after $\varphi_M(m_1) \leq \varphi_M(m_2)$. And another time, we have $\theta_M(m_1) \geq \theta_M(m_2)$.

Therefore, $\eta_M(l_1) = \eta_M(f(l_1)) = g(\eta_L(l_1)) \sqsubseteq g(\eta_L(l_2)) = \eta_M(m_2)$, and λ_M necessirly be a tri-value non-decreasing soft fuzzy left-right ideals over P as well. □

5 Tri-value soft fuzzy leftright ideals on decision making

The approximate function of an $\lambda-$set is a tri-value fuzzy set. The λ_{aggr} on tri-value fuzzy set is an operation by way of many approximate functions of an λ-set are jointed an only tri-value fuzzy set that is the aggregate tri-value fuzzy set of the λ-set.

Definition 5.1. Let $\lambda_L \in \lambda(C)$ and TVF($\mathbb{F}$) be the set of all tri-value fuzzy sets over $\mathbb{F}$. Then $\lambda-$aggregation operator, represented by λ_{aggr} is stated by

$$\lambda_{aggr} = TVF(\mathbb{F}) \times \lambda(C) \to TVF(C)$$

$$\lambda_{aggr}(L, \lambda_L) = \lambda_L* \tag{5.1}$$

$$where \quad \lambda_L* = \{(c, (\varphi^*_{\lambda_L}(c)))/c \in C\} \tag{5.2}$$

Which is an Tri-value fuzzy set over C. The value λ_L* is said to be aggregate Tri-value fuzzy set of λ_L. Here the degree of membership $\varphi_{\lambda_L} * (c)$ and degree of non membership $\theta_{\lambda_L} * (c)$ of "c" is stated as below:

$$\varphi_{\lambda_L} * (c) = 1/|\mathbb{F}| \sum_{l \in \mathbb{F}} \varphi_L(c)\varphi_{\eta_L}(c)$$

$$\theta_{\lambda_L} * (c) = 1/|\mathbb{F}| \sum_{l \in \mathbb{F}} \theta_L(c)\theta_{\eta_L}(c) \tag{5.3}$$

Here $|\mathbb{F}|$ is a number of members of $\mathbb{F}$.

Theorem 5.1. *Let $\lambda_L = \{(l, \varphi_L(l), \theta_L(l), \eta_L(l)) : l \in \mathbb{F},\ \eta_L(l) \in TVF(H)\}$ tri-value soft fuzzy LRI over H. Then the Tri-valued aggregate fuzzy set $\lambda*$ of λ_L is a tri-value fuzzy left right ideals of H.*

Proof. For any $l \in \mathbb{F}$ is a tri-value fuzzy left rigt ideal of H. That is $\forall\, h_1, h_2 \in H$, $\varphi_{\eta_L(l)}(h_1 \oplus h_2) \geq \inf\{\varphi_{\eta_L(l)}(h_1), \varphi_{\eta_L(l)}(h_2)\}$, $\varphi_{\eta_L(l)}(h_1 \otimes h_2) \geq \sup\{\varphi_{\eta_L(l)}(h_1), \varphi_{\eta_L(l)}(h_2)\}$ and for every $x, y, h, n \in H$, $h \oplus x \otimes n = y \oplus n$ implies $\varphi_{\eta_L(l)}(h) \geq \inf\{\varphi_{\eta_L(l)}(x), \varphi_{\eta_L(l)}(y)\}$. Then

$$\begin{aligned}
\varphi_{\eta_L*}(h_1 \oplus h_2) &= 1/|\mathbb{F}| \sum_{l \in \mathbb{F}} \varphi_L(l)(h_1 \oplus h_2) \\
&\geq \{1/|\mathbb{F}| \sum_{l \in \mathbb{F}} \varphi_L(l)(h_1), 1/|\mathbb{F}| \sum_{l \in \mathbb{F}} \varphi_L(l)(h_2)\} \\
&= \inf\{\varphi_{\eta_L*}(h_1), \varphi_{\eta_L*}(h_2)\} \qquad (5.4)
\end{aligned}$$

in similar way, we can find: $\varphi_{\eta_L*}(h_1 \otimes h_2) \geq \sup\{\varphi_{\eta_L*}(h_1), \varphi_{\eta_L*}(h_2)\}$, $\varphi_{\eta_L*}(h) \geq \inf\{\varphi_{\eta_L*}(x), \varphi_{\eta_L*}(y)\}$
$\theta_{\eta_L*}(h_1 \oplus h_2) \leq \sup\{\theta_{\eta_L*}(h_1), \theta_{\eta_L*}(h_2)\}$, $\theta_{\eta_L*}(h_1 \otimes h_2) \leq \inf\{\theta_{\eta_L*}(h_1), \varphi_{\eta_L*}(h_2)\}$
$\theta_{\eta_L*}(h) \leq \sup\{\theta_{\eta_L*}(x), \theta_{\eta_L*}(y)\}$, which gives λ_L* is an Tri-value fuzzy left right ideals of H. □

Notation: The notation λ_L*, tri-value aggregative fuzzy left right ideals tri-value fuzzy soft left right ideals of λ_L.

Example 5.1. Let H be an hemiring matrix , written by $H_{n\times n}$, X and Y is triangular and diagonal matrix. And let $\mathbb{F} = \{x, y\}$ the factors x and y stand for "triangular and diagonal". If L is an Tri-value fuzzy set over $\mathbb{F}$ stated by

$$\begin{aligned}
\varphi_L(l) &= \begin{cases} \frac{6}{10}, & \text{if } l = x \\ \frac{2}{10}, & \text{if } l = y \end{cases} \\
\theta_L(l) &= 1 - \varphi_L(l) \qquad (5.5)
\end{aligned}$$

$$\begin{aligned}
Define,\ \eta_L\ by\ \varphi_{\eta_L(x)}(t) &= \begin{cases} \frac{0}{10}, & t\ is\ non\ triangular\ matrix \\ \frac{10}{10}, & t\ is\ triangular\ matrix \end{cases} \\
\varphi_{\eta_L(x)}(t) &= \begin{cases} \frac{0}{10}, & t\ is\ non\ diagonal\ matrix \\ \frac{10}{10}, & t\ is\ diagonal\ matrix \end{cases} \\
\theta_{\eta_L(x)}(t) &= 1 - \varphi_{\eta_L(x)}(t) \qquad (5.6)
\end{aligned}$$

We can confirm that λ_L* is said to be Tri-value aggregate fuzzy ideal of λ_L.
For Example 5.1, we assume an Tri-value fuzzy left right ideals of λ_L. So we can yield into respect the common condition λ_L*.

Definition 5.2. We can create decision taking technique by the following rules.
Rule 1 Create an $\lambda-$set over C.
Rule 2 Obtain tri-value aggregate fuzzy set λ_L* of λ_L.
Rule 3 Obtain $\sup(i) = \sup\{\varphi_{\lambda_L*}(i)/i \in C\}$ and $\inf(i) = \inf\{\theta_{\lambda_L*}(j)/j \in C\}$
Rule 4 Obtain $\epsilon \in [0,1]$ such that $(i, \sup(i), \epsilon) \in \lambda_L*$ and $\delta \in [0,1]$ such that $(j, \inf(j), \delta) \in \lambda_L*$.
Rule 5 Obtain $\frac{\sup(i)}{\sup(i)+\epsilon} = \epsilon'$ and $\frac{\delta}{\inf(i)+\delta} = \delta'$
Rule 6 Appropriate element of C is represented by Appr(i) and is selected as follows:

$$Appr(i) = \begin{cases} i, & \epsilon' > \delta' \\ j, & \epsilon' < \delta' \end{cases}$$

Example 5.2. We provide λ-decision taking application.
Consider the person sweepstakes coupons, were assume only 5 digts , and they are stated as $H = \mathbb{Q} = \{0,1,2,3,4\}$. The purchaser assumes a set of factors $\mathbb{F} = \{l_1, l_2, l_3\}$. For $k = 1,2,3$ the factors l_k position for "Liking", "continuously happening" and "mutual meddling", respectively. After argument, each dight from point of sight of the drop a foal and the limitation according to decision subset $L = \{(l_1, (\frac{6}{10}, \frac{3}{10})), (l_2, (\frac{8}{10}, \frac{2}{10})), (l_3, (\frac{7}{10}, \frac{2}{10}))\}$ of $\mathbb{F}$. Lastly, the purchers constructs the next $\lambda-$set over H.

$$\lambda_L = \Bigg\{ \Bigg[\left(l_1, \left(\frac{6}{10}, \frac{3}{10}\right)\right), \Bigg\{ \left(0, \left(\frac{2}{10}, \frac{6}{10}\right)\right), \left(1, \left(\frac{7}{10}, \frac{3}{10}\right)\right), \left(2, \left(\frac{4}{10}, \frac{5}{10}\right)\right), \left(3, \left(\frac{8}{10}, \frac{1}{10}\right)\right), \left(4, \left(\frac{1}{10}, \frac{6}{10}\right)\right) \Bigg\} \Bigg],$$
$$\Bigg[\left(l_2, \left(\frac{8}{10}, \frac{2}{10}\right)\right), \Bigg\{ \left(0, \left(\frac{4}{10}, \frac{6}{10}\right)\right), \left(1, \left(\frac{9}{10}, \frac{1}{10}\right)\right), \left(2, \left(\frac{5}{10}, \frac{4}{10}\right)\right), \left(3, \left(\frac{6}{10}, \frac{2}{10}\right)\right), \left(4, \left(\frac{6}{10}, \frac{3}{10}\right)\right) \Bigg\} \Bigg],$$
$$\Bigg[\left(l_3, \left(\frac{7}{10}, \frac{2}{10}\right)\right), \Bigg\{ \left(0, \left(\frac{2}{10}, \frac{6}{10}\right)\right), \left(1, \left(\frac{6}{10}, \frac{3}{10}\right)\right), \left(2, \left(\frac{7}{10}, \frac{1}{10}\right)\right), \left(3, \left(\frac{7}{10}, \frac{2}{10}\right)\right), \left(4, \left(\frac{6}{10}, \frac{3}{10}\right)\right) \Bigg\} \Bigg] \Bigg\}$$

(i) The aggregate Tri-value fuzzy set cab be set up as
(ii) $\lambda_L* = \left\{ (0, (\frac{193}{100}, \frac{14}{100})), (1, (\frac{52}{100}, \frac{5}{100})), (2, (\frac{37}{100}, \frac{8}{100})), (3, (\frac{48}{100}, \frac{3}{100})), (4, (\frac{32}{100}, \frac{10}{100})) \right\}$
(iii) $\sup(i) = \frac{52}{100}$ and $\inf(j) = \frac{3}{100}$
(iv) $(1, (\frac{52}{100}, \frac{5}{100})) \in \lambda_L*$ and $(3, (\frac{48}{100}, \frac{3}{100})) \in \lambda_L*$
(v) $\epsilon' = \frac{\frac{52}{100}}{\frac{52}{100} + \frac{5}{100}} = \frac{90}{100}$ and $\delta' = \frac{\frac{48}{100}}{\frac{48}{100} + \frac{3}{100}} = \frac{93}{100}$
(vi) Since, $\epsilon' < \delta'$, $Appr(i) = 3$.

We discuss the decision-taking use of the 0 to 4 number digit only. In the same way, we can apply many number digit from 0 to 9.

6 Conclusion

The paper presented a few basic knowledge of tri-value fuzzy soft left-right ideals. And we stated tri-value fuzzy soft homomorphism, tri-value equivalent fuzzy soft ideals, and tri-value non-decreasing fuzzy soft left-right ideals and some properties of them have been given. Lastly, we examined aggregate tri-value fuzzy left-right ideals of hemiring based on decision taking.

Conflict of Interests

The authors declare that there is no conflict of interest regarding the publication of this paper.

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